Platform Competition with Network-based Advertising

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- In late 2015, Twitter changed its advertising strategy (Kafka 2016)
- Users with many followers no longer saw ads, or saw very few
- Likely an attempt to retain influential users
  - Risk that influential users move to another platform, like Instagram
  - Influential users engaged $\longrightarrow$ Followers engaged

- Users on Twitter/Instagram care about
  1. Viewing posts (especially from friends)
  2. Not seeing ads
- Both platforms are free, so cannot compete on price
- Can compete through advertising
  - Choose **ad load** for each user: ratio of ads to real posts

Updates

- Simplified model $\longrightarrow$ more tractable

- N consumers linked in a network
- Two platforms, labeled 0 and 1
- Network modeled as a graph, adjacency matrix $G = (g_{ij})$
    - **Exogenous** network (for now)

- Per-period utility for consumer i spending t minutes on platform m:

$$\zeta_i^m + \underbrace{(1 - p_i^m)t - \frac{1}{2}t^2}_{\text{Content/ads}} + \underbrace{t\nu \sum_{j=1}^{N} g_{ij}\chi_j^m}_{\text{Network effects}}$$

- ζ_i^m: platform-specific benefit
- p_i^m: ad load for consumer i on platform m
- g_{ij}: weight on link from consumer i to consumer j
- χ_j^m: indicates whether consumer j is on platform m
- ν: strength of network effects
- See e.g. Chen, Zenou, and Zhou 2018

- Timing each period:
  1. Platforms set ad loads given current platform choices $x = (x_1, \ldots, x_N)$
  2. One consumer randomly chosen to update platform choice
     - Draw $\zeta_i^0 - \zeta_i^1$ from distribution Φ
     - Choose platform
     - No multihoming (for now)
  3. Each consumer chooses how much time to spend on their platform this period
  4. Platforms and firms receive payoffs
- Optimal number of minutes for consumer i to spend on platform m:

$$t_i^* = 1 - p_i^m + \nu \sum_{j=1}^{N} g_{ij} \chi_j^m$$

Consumer i, if selected to update, chooses platform 0 when

$$\underbrace{\zeta_i^0 + \frac{1}{2}\left(1 - p_i^0 + \nu \sum_{j=1}^{N} g_{ij}(1 - x_j)\right)^2}_{\text{Utility from platform 0}} > \underbrace{\zeta_i^1 + \frac{1}{2}\left(1 - p_i^1 + \nu \sum_{j=1}^{N} g_{ij} x_j\right)^2}_{\text{Utility from platform 1}}$$

$$\implies \zeta_i^0 - \zeta_i^1 > \frac{1}{2}\left(1 - p_i^1 + \nu \sum_{j=1}^{N} g_{ij} x_j\right)^2 - \frac{1}{2}\left(1 - p_i^0 + \nu \sum_{j=1}^{N} g_{ij}(1 - x_j)\right)^2$$

Consumer i chooses platform 0 with probability

$$q(i,x) := 1 - \Phi\left[\frac{1}{2}\left(1 - p_i^1 + \nu \sum_{j=1}^{N} g_{ij} x_j\right)^2 - \frac{1}{2}\left(1 - p_i^0 + \nu \sum_{j=1}^{N} g_{ij}(1 - x_j)\right)^2\right]$$

- Each period, platform m receives αp_i^m from each consumer i on platform m
  - Previously $t_i^* p_i^m$
  - Brands pay market rate α for advertising space on the platform
  - Platform can increase ad load costlessly
  - Time consumer spends on platform doesn't affect payment to platform
- This is a big simplification
  - On Instagram, advertisers set a budget and duration for each ad they want to run
  - Then, Instagram's algorithm shows the ad to users
- Platforms set ad loads to maximize expected payoffs

- x: the state (platform choices of all consumers)
- δ: discount rate
- Value function for platform 0:

$$v^0(x) = \sum_{i=1}^{N} \frac{1}{N} \underbrace{q(i,x)\alpha p_i^0}_{\text{Expected payoff if consumer i selected}} + \frac{N-1}{N} \underbrace{(1-x_i)\alpha p_i^0}_{\text{Expected payoff if consumer i not selected}}$$
$$+\delta \sum_{i=1}^{N} \frac{1}{N} \left(q(i,x) v^0 \underbrace{[(I-E_{ii})x]}_{\text{New state if i chooses 0}} + (1-q(i,x)) v^0 \underbrace{[(I-E_{ii})x + e_i]}_{\text{New state if i chooses 1}} \right)$$

Value function for platform 1:

$$v^1(x) = \sum_{i=1}^{N} \frac{1}{N} \underbrace{(1 - q(i,x))\alpha p_i^1}_{\text{Expected payoff if consumer i selected}} + \frac{N-1}{N} \underbrace{x_i \alpha p_i^1}_{\text{Expected payoff if consumer i not selected}}$$

$$+\delta \sum_{i=1}^{N} \frac{1}{N} \left(q(i,x) v^1 \underbrace{[(I - E_{ii})x]}_{\text{New state if i chooses 0}} + (1 - q(i,x)) v^1 \underbrace{[(I - E_{ii})x + e_i]}_{\text{New state if i chooses 1}} \right)$$

FOC(p_i^0):

$$
\begin{aligned}
0 =& \frac{1}{N}\frac{\partial q}{\partial p_i^0}\alpha p_i^0 + \frac{1}{N}\alpha q(i,x) + \frac{N-1}{N}(1-x_i)\alpha \\
&+ \delta\frac{1}{N}\frac{\partial q}{\partial p_i^0}v^0[(I-E_{ii})x] + \delta\frac{1}{N}q(i,x)\underbrace{\frac{\partial v^0[(I-E_{ii})x]}{\partial p_i^0}}_{\text{zero in MPE}} \\
&+ \delta\frac{1}{N}\left(-\frac{\partial q}{\partial p_i^0}\right)v^0[(I-E_{ii})x+e_i] + \delta\frac{1}{N}(1-q(i,x))\underbrace{\frac{\partial v^0[(I-E_{ii})x+e_i]}{\partial p_i^0}}_{\text{zero in MPE}}
\end{aligned}
$$

FOC(p_i^1):

$$\begin{aligned}
0 = &- \frac{1}{N}\frac{\partial q}{\partial p_i^1}\alpha p_i^1 + \frac{1}{N}\alpha(1-q(i,x)) + \frac{N-1}{N}x_i\alpha \\
&+ \delta\frac{1}{N}\frac{\partial q}{\partial p_i^1}v^1[(I-E_{ii})x] + \delta\frac{1}{N}q(i,x)\underbrace{\frac{\partial v^1[(I-E_{ii})x]}{\partial p_i^1}}_{\text{zero in MPE}} \\
&+ \delta\frac{1}{N}\left(-\frac{\partial q}{\partial p_i^1}\right)v^1[(I-E_{ii})x+e_i] + \delta\frac{1}{N}(1-q(i,x))\underbrace{\frac{\partial v^1[(I-E_{ii})x+e_i]}{\partial p_i^1}}_{\text{zero in MPE}}
\end{aligned}$$

First order conditions

$$p_i^0 = \frac{(N-1)(x_i - 1) - q(i,x)}{\frac{\partial q(i,x)}{\partial p_i^0}} + \frac{\delta}{\alpha}\left\{v^0[(I - E_{ii})x + e_i] - v^0[(I - E_{ii})x]\right\}$$

$$p_i^1 = \frac{(N-1)x_i + (1 - q(i,x))}{\frac{\partial q(i,x)}{\partial p_i^1}} + \frac{\delta}{\alpha}\left\{v^1[(I - E_{ii})x] - v^1[(I - E_{ii})x + e_i]\right\}$$

$$v^0(\mathbf{0}) = -\frac{1}{2}\frac{\alpha}{1+\delta}\frac{(q(1,\mathbf{0})+1)^2}{\frac{\partial q(1,\mathbf{0})}{\partial p_1^0}} - \frac{1}{2}\frac{\alpha}{1+\delta}\frac{(q(2,\mathbf{0})+1)^2}{\frac{\partial q(2,\mathbf{0})}{\partial p_2^0}} + \frac{\delta}{1+\delta}v^0(e_1) + \frac{\delta}{1+\delta}v^0(e_2)$$

$$v^0(e_1) = -\frac{1}{2}\alpha\frac{\left(q(2,e_1)+1\right)^2}{\frac{\partial q(2,e_1)}{\partial p_2^0}} - \frac{1}{2}\alpha\frac{\left(q(1,e_1)\right)^2}{\frac{\partial q(1,e_1)}{\partial p_1^0}} + \delta v^0(\mathbf{1})$$

$$v^0(e_2) = -\frac{1}{2}\alpha\frac{\left(q(1,e_2)+1\right)^2}{\frac{\partial q(1,e_2)}{\partial p_1^0}} - \frac{1}{2}\alpha\frac{\left(q(2,e_2)\right)^2}{\frac{\partial q(2,e_2)}{\partial p_2^0}} + \delta v^0(\mathbf{1})$$

$$v^0(\mathbf{1}) = -\frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(1,\mathbf{1}))^2}{\frac{\partial q(1,\mathbf{1})}{\partial p_1^0}} - \frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(2,\mathbf{1})^2}{\frac{\partial q(2,\mathbf{1})}{\partial p_2^0}}$$

N=2 case

$$
\begin{aligned}
v^0(\mathbf{0}) = & -\frac{1}{2}\frac{\alpha}{1-\delta}\left(\frac{q(1,\mathbf{0})^2}{\frac{\partial q(1,\mathbf{0})}{\partial p_1^0}} + \frac{q(2,\mathbf{0})^2}{\frac{\partial q(2,\mathbf{0})}{\partial p_2^0}}\right) \\
& -\alpha\left(\frac{q(1,\mathbf{0})}{\frac{\partial q(1,\mathbf{0})}{\partial p_1^0}} + \frac{q(2,\mathbf{0})}{\frac{\partial q(2,\mathbf{0})}{\partial p_2^0}}\right) \\
& -\frac{1}{2}\alpha\left(\frac{1}{\frac{\partial q(1,\mathbf{0})}{\partial p_1^0}} + \frac{1}{\frac{\partial q(2,\mathbf{0})}{\partial p_2^0}}\right)
\end{aligned}
$$

$$v^0(e_1) = -\frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(1,e_1)^2}{\frac{\partial q(1,e_1)}{\partial p_1^0}} - \frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(2,e_1)^2}{\frac{\partial q(2,e_1)}{\partial p_2^0}} - \alpha\frac{q(2,e_1)}{\frac{\partial q(2,e_1)}{\partial p_2^0}} - \frac{1}{2}\alpha\frac{1}{\frac{\partial q(2,e_1)}{\partial p_2^0}}$$

$$v^0(e_2) = -\frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(1,e_2)^2}{\frac{\partial q(1,e_2)}{\partial p_1^0}} - \frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(2,e_2)^2}{\frac{\partial q(2,e_2)}{\partial p_2^0}} - \alpha\frac{q(1,e_2)}{\frac{\partial q(1,e_2)}{\partial p_1^0}} - \frac{1}{2}\alpha\frac{1}{\frac{\partial q(1,e_2)}{\partial p_1^0}}$$

$$v^0(\mathbf{1}) = -\frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(1,\mathbf{1})^2}{\frac{\partial q(1,\mathbf{1})}{\partial p_1^0}} - \frac{1}{2}\frac{\alpha}{1-\delta}\frac{q(2,\mathbf{1})^2}{\frac{\partial q(2,\mathbf{1})}{\partial p_2^0}}$$

1. Numerically solve for prices
   - Can calculate value function: e.g. start with $v^0(\mathbf{1})$ and work backwards
2. Choose consumer to update
3. Given prices, they may or may not switch platforms
4. Repeat using new state